Fingerprint Smoothing Using Different Interpolation Techniques

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Abstract

Classical fingerprint analysts use binary images in the minutiae recognition and extraction processes. These obtained images do not have a sufficient quality that allows the extraction of the robust primitives, either during the contours detection or during skeletonization. In this study, a mathematical approach for the fingerprint curves modeling have been performed. The adopted smoothing technique is based on two geometric interpolation types adapted to fingerprint images. The obtained results reveal that the Bezier curve method has an error lower than that by the cubic spline method. The Bézier curve method has a RMS value of 0.035 pixels and an average maximum error of the order of 0.33 pixels. On the other hand, the cubic spline method has a RMS value about of 0.043 pixels and an average maximum error about of 0.37 pixels. We can see that the proposed method facilitates the design process of real-world objects and makes the fingerprint curves smooth.

Keywords:

Fingerprint image; Bézier curve method; Cubic spline method; Gabor filter; smooth curve.

1. Introduction

Biometrics is a method of identifying individuals either by their physical characteristics such as fingerprints, face, and iris or by their behavioral characteristics such as speech. Despite the evolution of new biometric methods considered as more reliable and more robust, the fingerprint identification remains the most widely used method (Soundharadevi and Pushparani, 2016- Conti et al., 2017). Indeed, the fingerprint offers an effective solution to identify people and it is very easy to acquire. In addition, this biometric technology is considered one of the cheapest in terms of cost.

Understanding the structure of fingerprint curves plays a capital role in the minutiae recognition and extraction process. Usually, conventional fingerprint-based recognition techniques go through the binarization and skeletonization step to extract the minutiae existing in the fingerprint image (Saleh et al., 2011- Nagar et al., 2010). Generally, the bending edges state is also an important fingerprint characteristic. However, false curves are generated if the image quality is poor and it can be observed that the bending curve tendency is irregular. On the basis of these observations, we propose in this paper a new algorithm for smoothing these curves. In this algorithm, a pre-processing and fingerprint image enhancement step is performed to simplify the curve detection and minutiae extraction task. This phase requires using the Gober filter to improve the image (Sojan and Kulkarni, 2016, Mohammedsayeem-uddin et al. 2014, Hussain et al., 2016), and then the conversion of the fingerprint image into a binary image (Shetter et al., 2018- Rani and Kothuru, 2017). The next step requires the implementation of the skeletonization algorithm to reduce the center-line thickness to one pixel (Karani and Aithal, 2017, Bataineh, 2018, Leslie and Sumathi, 2018). Then, a 3*3 matrix is used to extract the minutiae types (Jothi and Palanisamy, 2016, Ain et al., 2018, Krish et al., 2019). Based on the curves and minutiae detected, we introduce a fine-tuning algorithm that automatically detects the control points and uses them as nodes for curve interpolation. Finally, two interpolation types are implemented to smooth the fingerprint curves and assess the performance of the considered algorithm.

In this paper, an approach to smooth the fingerprint curves is developed. In section 2, a brief description of related works is given. In section 3, we provide an expression of a potentials idea adapted to our problem. In Section 4, we present the segmentation technique of fingerprint images used and the minutiae detection method. Then, section 5 describes the theoretical aspect for the two interpolations types used. The experimental results and the error calculation are presented in sections 6. Finally, we finish with the discussion and the conclusion in sections 7 and 8, respectively.

2. Related Works

Over the years, many interpolation methods such as the polynomial interpolation, linear or cubic spline methods, least squares method, fractal interpolation and Bezier curve method have been applied to smooth curves in different domains and to facilitate the design process of real-world objects. Ballan M. (1998) has used the smoothing, classification and fingerprints identification on the basis of singular points (delta and core points) to introduce the directional fingerprints processing. He has used the directional histograms of a fingerprint to detect delta and core points. His proposed algorithm involves directional image representation as well as singular point detection. Perumal and Ramaswamy, 2009 have proposed an innovative and efficient scheme for the fingerprint images compression. They have used representations of the Bézier curve to achieve this aim. Their proposed technique involves two major steps; the first step is to extract the peaks present in the fingerprint image with their coordinate values. Thereafter, they have used the Bézier curve technique to detect the control points. These control points have been used to reconstruct the image of the fingerprint using the Bézier curves. Vijayaragavan et al., 2014 have suggested a model to compare the knowledge from the signature. They have used the Bézier curve algorithm to categorize points on the curve and they have used the signature behaviors for verification. Bezier curves are segmented from the signature by examining the pixel color. Chong et al., 1992 have proposed a data compression method for digitized fingerprint images based on the B-spline functions. They have developed algorithms for extracting, classifying or recognizing fingerprint characteristics in the first instance. Then, they have given a description of two B-spline representation approaches for compressing data from a fingerprint image. Guedri et al., 2017a have proposed a mathematical approach for 2D reconstruction of the retinal vascular tree. They have extracted the graph of unsupervised topology of the blood vessel from the image of the human retina. They have used three types of interpolations, i.e. the least squares method, linear spline method and cubic spline method, to smooth the blood vessel curves. Guedri et al., 2017b have presented a method to avoid the major disadvantage of skeletonization of the human retina image. They have used, in a first phase, all the segmentation steps to detect the blood vessels curve including image binarization, skeletonization and finally the step for location identification of characteristic points (endpoints, middle-point, and bifurcation-points). In a second phase, they have used the B-Spline interpolation method to improve the blood vessels curvature quality and to obtain smoother curves and more realistic effects.

3. Problem proposal

The main objective of this work is to eliminate the negative effect of digital image processing in the study of fingerprint image, such as the irregularities and singularities of a curve from the obtained skeletal images. After using the Gober technique to improve the image quality, we use a binarization technique to obtain a binary image; we can extract the central-line by the skeletonization technique. But, the drawback of this approach is that the fingerprint curves connectivity is not realistic, as shown in Figure 1.

Fig. 1. The negative effect of digital image

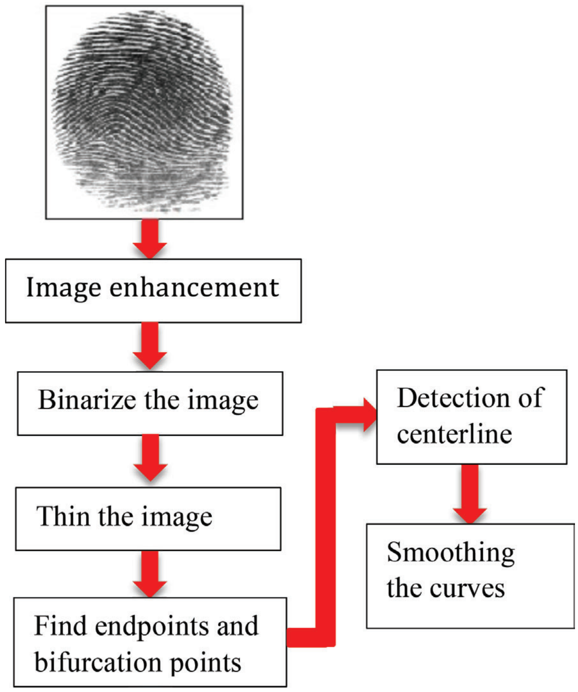
The proposed method is used to reconstruct the most authentic fingerprint curves and at the same time to mitigate the drawbacks of digital image processing. The first objective is to enhance the acquired image quality. Subsequently, we elicit data from image by segmentation techniques such as binarization as the first task. Then, the image skeletonization is the second task and finally extraction of the minutiae and the fingerprint curves. The final step in the proposed algorithm is to use two interpolation types to get a smooth center-line closer to nature. Figure 2 shows the block diagram of this approach.

4. Image Pre-processing

4.1. Fingerprint image enhancement

First, fingerprint images are enhanced based on Gabor filters as proposed in Sojan and Kulkarni, 2016. The filter properties that it is at the same time selective in frequency and orientation, moreover it has an optimal joint resolution in the space and frequency domains (Mohammedsayeemuddin et al. 2014- Hussain et al., 2016). Let the fingerprint image be represented by I (x, y). The symmetrical two-dimensional Gabor filter is given by:

Fig. 2. Block diagram of the proposed method



$$g(x,y:\theta,f)=\exp\left\{-\frac{x_\theta^{2}}{\sigma_x^{2}}+\frac{y_\theta^{2}}{\sigma_y^{2}}\right\}.\cos\left(2\pi f.x_\theta\right) \quad (1)$$

where θ is the filter orientation, [xθ, yθ] are the coordinates of [x, y] after a clockwise Cartesian axes rotation the by an angle (90°-θ), as shown by the following Eq. 2:

$$\begin{bmatrix} x_\theta \\ y_\theta \end{bmatrix}=\begin{bmatrix} \cos(90-\theta) & \sin(90-\theta) \\ -\sin(90-\theta) & \cos(90-\theta) \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$=\begin{bmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} \quad (2)$$

In the Eq. (1), f is the frequency of the sinusoidal plane wave in the direction θ from the x axis, and σx and σy are the space constants of the Gaussian envelope along the x and y axes. Four values of θ, namely 0, 45, 90, 135 degrees, are used to obtain four different filters. These filters are convolved with the image I (x, y) to obtain the corresponding filtered outputs given by Eθ (x, y) = g (x, y, f, θ) * I (x, y) (Sojan and Kulkarni, 2016).

4.2. Binarization

This phase focuses on the images segmentation and seeks an effective method to clearly separate the background and the object. In other words, it is about finding a binarization method that can effectively determine the threshold for each image point. Binarization consists of transforming a multi-bit pixel into a 1-bit image (Shetter et al., 2018- Rani and Kothuru, 2017). For that, we will do a thresholding. If the pixel value is below the threshold; we associate it with the value 0. If the pixel value is equal to or greater than the threshold we associate it with the value 1.

4.3. Skeletonization

The skeleton is a compact and efficient representation of the form. It is widely used in the image analysis field; the skeleton represents the line centered in the form. Thinning is the chosen skeletonization method. Thinning is a method of "peeling" the shape until one gets a connected point set of a pixel wide that retains the shape topology (Karani and Aithal, 2017). In other words, the principle of this method is to examine, in a predetermined order, the form contour points in order to delete them iteratively (Bataineh, 2018- Leslie and Sumathi, 2018). The contour points are then marked for deletion after the identification iteration. These two phases avoid deleting a skeleton complete branch in iteration.

4.4. Minutiae extraction

In this method, a skeleton image is used for minutiae extraction (Jothi and Palanisamy, 2016, Ain et al., 2018, Krish et al., 2019). It involves the use of a 3x3 window to check the neighboring area of each image pixel, as shown in Figure 3. A pixel is classified as an isolated point if it has no neighboring pixel, and as an endpoint if it has a single neighboring pixel in the image, and as a continuing ridge point if it has 2 neighboring pixels, and as a bifurcation point if it has 3 neighboring pixels, and as a crossing point if it has 4 neighboring pixels (Ain et al., 2018).

Fig. 3. The 8 Neighborhoods

P1	P2	P3
P4	P	P5
P6	P7	P8

5. Interpolation

Let given dataset points

$$P\binom{x_i}{y_i} = \{(x_0, y_0);\ (x_1, y_1);\ \dots\ ;\ (x_n, y_n)\} \in \mathbb{R}^2.$$

The purpose of the polynomial interpolation is to find a polynomial that passes through all data points (Roy et al., 2013).

The main purpose of the interpolation is to find a function to approximate the functional values and the data of the original values (Parsania and Virparia, 2016). The interpolation function usually goes through the original dataset, with the curve point adjustment; we just want a function that fits well into the original data points. With curve fitting, the approximation function does not have to go through the original data set.

5.1. Bezier Curve

Bézier curves are parametric polynomial curves described by Pierre Bézier in 1962. They are used to design auto parts (Gousenbourg et al., 2016- Fierz, 2018). They are also used in several applications such as image synthesis, font rendering, animation, environment design and robotics (Baydas and Karakas, 2019).

Let (Pi = (xi, yi), i = 0, 1, 2, ..., n) be the control points of the Bezier curve. The Bezier curve of degree n can be defined by the following Eq. 3 (Gousenbourg et al., 2016):

$$\begin{cases} x(t) = \sum_{i=0}^{n} x_i B_i(t) \\ y(t) = \sum_{i=0}^{n} y_i B_i(t) \end{cases} \qquad 0 \le t \le 1 \qquad (3)$$

with (x(t),y(t)) are the points on the curve, B_i (t) (i=0,1,…,n) are the Bernstein polynomials. They are used as the basis functions, for polynomial order n. The ith basis function is defined by the following Eq. 4:

$$B_i(t) = \frac{n!}{i!\,(n-i)!} t^i (1-t)^{n-1} \qquad (4)$$

The main advantage of Bezier curves is that the number of control points can be very large without the curve becoming impossible to manipulate (Fierz, 2018- Baydas and Karakas, 2019). It also allows to have interpolation points in the middle of the curve and not only at the ends.

5.2. Interpolation by cubic splines

The cubic spline interpolation is a piecewise interpolation, let y (x) over an interval [x0, xn] that has been partitioned into subintervals [xi-1, xi], i = 1,2, ..., n (Abdul-Karim et al., 2018).

This interpolation type consists in replacing, on each subinterval, the function y by a third degree polynomial (Yaghoobi et al., 2017- Wu et al., 2015), so that the interpolating function is continuous, as well as its first and second derivatives on the whole interval [x0, xn] (Parveen and Tokas, 2015).

The interpolation cubes are defined by the following Eq. 5 (Wu et al., 2015) :

$$f_i(x) = a_i x^3 + b_i x^2 + c_i x + d_i \qquad (5)$$

In the above equation, x_(i-1)≤x≤x_i and i=1,2,…,n. Let use the notation for the second derivative: f_i^” (x_i)=f_i”.

Using the function continuity, after some algebraic manipulations, it is straightforward to obtain the following expression:

$$f_i(x) = f''_{i-1}\frac{(x_i - x)^3}{6h_i} + f''_i\frac{(x - x_{i-1})^3}{6h_i} + \left[\frac{y_{i-1}}{h_i} - f''_{i-1}\frac{h_i}{6}\right](x_i - x) + \left[\frac{y_i}{h_i} - f''_i\frac{h_i}{6}\right](x - x_{i-1})$$

The functions f_i (x) will be fully known after calculating values of f_i”.

To obtain these values, it is necessary to use the conditions of continuity of the first derivatives at the interior points:

$$f'_i(x_i) = f'_{i+1}(x_i) \quad , \quad i = 1,2,\ldots,n-1$$

We deduce, the following equation:

$$h_i f''_{i-1} + 2(h_i + h_{i+1}) f''_i + h_{i+1} f''_{i+1} = \frac{6}{h_{i+1}}(y_{i+1} - y_i) + \frac{6}{h_i}(y_{i-1} - y_i) \quad ; \quad i = 1,2,\ldots,n-1.$$

We thus obtain a linear system of (n-1) equations with n+1 unknown, the *fi”*.

There remains therefore the possibility of imposing two additional conditions, obtained for example by the interval boundary conditions at x_0 and x_n. We impose the following two conditions:

$$f''_1(x_0) = 0 \quad and \quad f''_n(x_n) = 0$$

The natural cubic splines are therefore obtained unambiguously.

The n-1 unknowns *f1^”,f2^”,…,fn”* are then solution of the linear system written above. This tridiagonal system is symmetrical. This system has a unique solution because its tri-diagonal matrix is diagonally dominant and is therefore invertible.

If the interpolation points are uniformly distributed on [x0, xn], all hi are equal and the system becomes, for i = 1, 2, ..., n-1:

$$f''_{i-1} + 4f''_i + f''_{i+1} = \frac{6}{h^2}\left[y_{i+1} - 2y_i + y_{i-1}\right]$$

5.3. Error calculations

The objective of this work is to achieve the fingerprint curves reconstruction by using two mathematical interpolations (Fortin et al., 2014, Brereton, 2018, Wang and Lu, 2018). Since it is impossible to arrive at exactly the same result, the experiment success must be evaluated by some comparisons between the obtained result values and the initial values. For this purpose, we use the root-mean-square error (RMS error) and the absolute value of the maximum error.

- **Root-mean-square error (RMS)**

This method is used to measure the difference between the initial data and the reconstructed data, as shown in Eq. 6 below (Fortin et al., 2014, Brereton, 2018):

$$RMS = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}\left(f_i - f_{i,ini}\right)^2} \qquad (6)$$

where N is the data number, f_i is the reconstructed data and f_(i,ini)i is the initial data.

Absolute value of the maximum error:

The absolute value of the maximum error between the reconstructed data and the original data is defined according the following equation (Wang and Lu, 2018- Al-Janabi et al., 2018):

$$MAE = \max_{i=1:N}\left|f_i - f_{i,ini}\right| \qquad (7)$$

6. Result

To evaluate our approach, the proposed algorithm is implemented on a workstation equipped with an Intel Pentium B960 processor at 2.20 GHz and 4 GB of RAM processor and Windows 7 OS, using the Matlab language. In this study, we use fingerprint images from the Fingerprint Verification Competition database (FVC2006). It contains four sub-databases, DB1, DB2 and DB3, which are acquired with different sensors, and B4 which is created with a synthetic generator. Fingerprint images have different sizes with a resolution of 500 dpi and they are devised in two sets:

- Set A: consisting of fingers numbered from one to 100

- Set B: made up of fingers numbered from 101 to 110 and made available to users.

6.1. Image segmentation

The fingerprint raw image is extracted from the database. Subsequently, to enhance these images quality, the Gabor filter is used, as shown in Fig. 4. Then, the thresholding method is used to transform the grayscale image into a binary image. We use a thinning algorithm to determine the fingerprint skeleton and detect the curves with a one-pixel width. The obtained image guarantees an efficient extraction of the minutia points. The scanning of the skeleton image with a 3 * 3 matrix makes it possible to detect each pixel type (Endpoint, Bifurcation point) as shown in Fig. 4 e and f.

6.2. Smoothing the fingerprint curve

After determining the central line positions and the extraction of the points of minutiae, we use two interpolations types, namely Bezier Curve and cubic splines, to reconstruct the smooth fingerprint curves. The obtained results are shown in Fig. 5:

Fig. 4. Image Pre-processing: (a) raw image, (b) enhanced image, (c) Binary image, (d) Skeletonization image, (e) Endpoints (Red points), (f) Bifurcation points (Blue points).

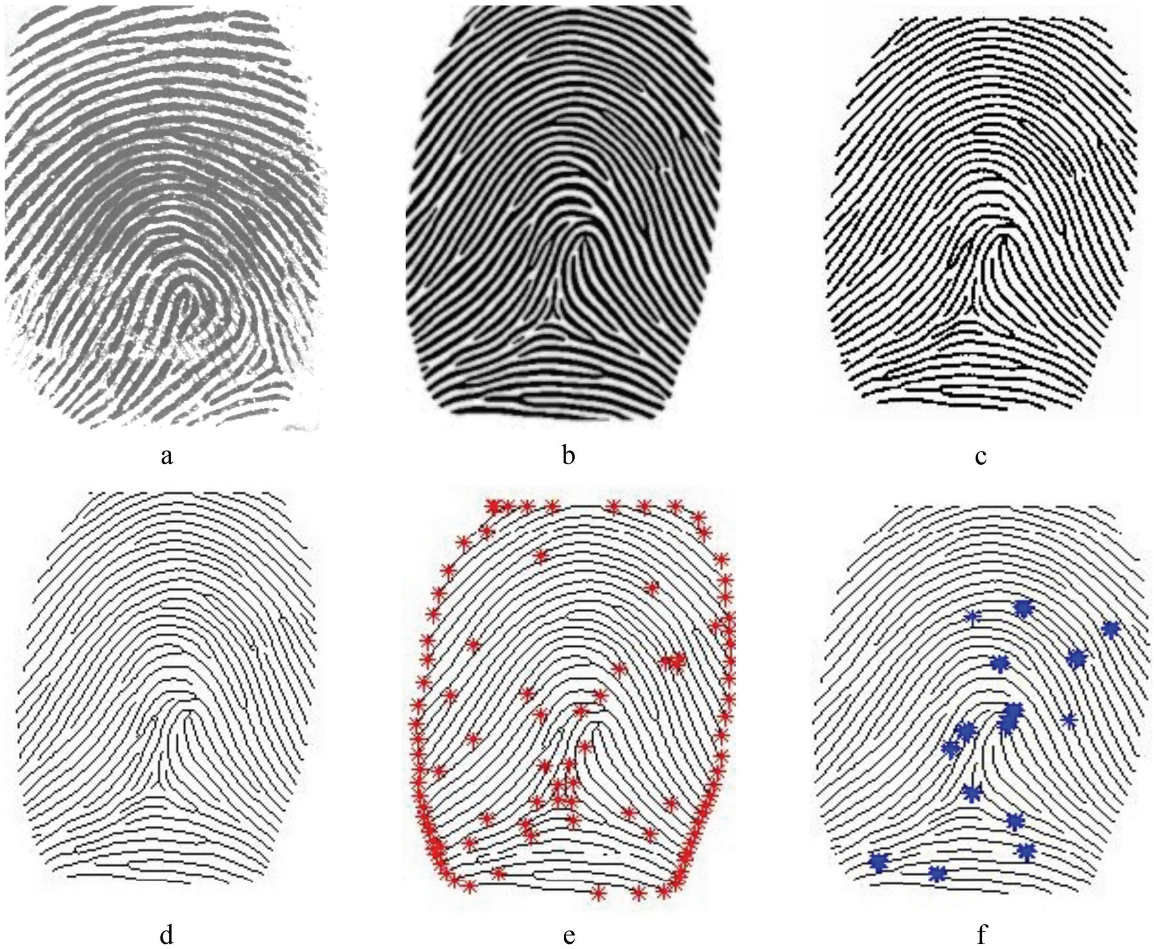


Fig. 5. Interpolation examples of a fingerprint curve, (a) original data, (b) Bezier Curve interpolation, (c) Cubic spline interpolation.

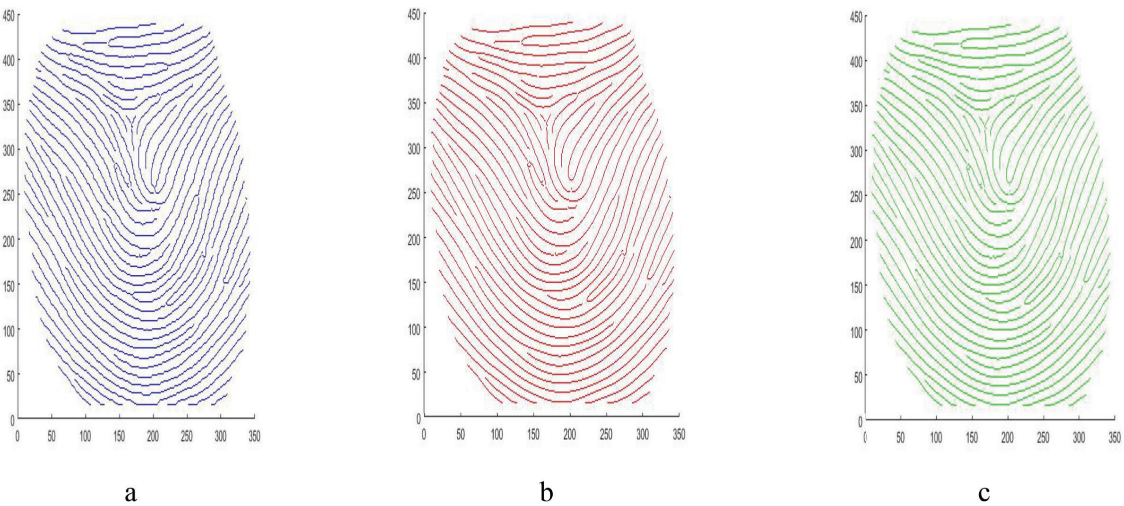


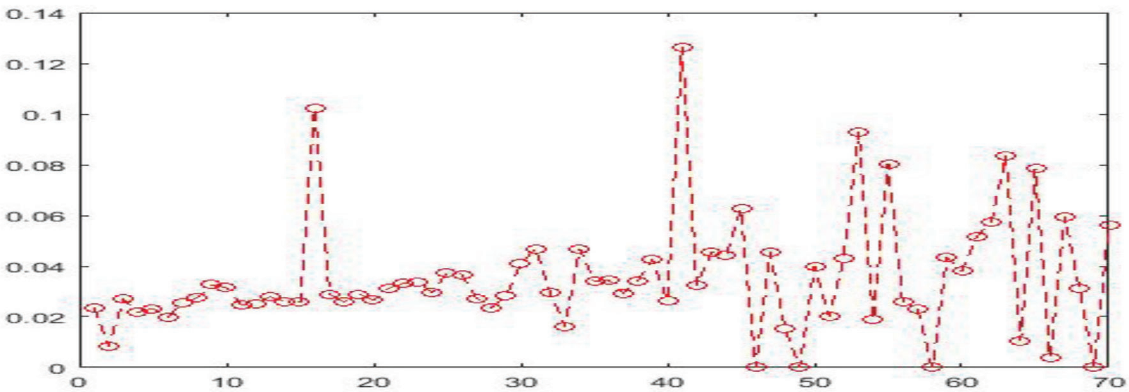
Figure 5 shows the typical processing results on a sample of fingerprint curves. Figure 5.b shows the interpolation result with the Bézier curve method and Figure 5.c with the cubic splines method. From these results, we can see that the curves are very close to each other. They are almost identical either for the curve using the interpolation Bezier Curve or for the second interpolation type i.e. interpolation by cubic splines. We can also notice that the two obtained curves have very realistic connectivity and are very close to the natural models.

6.3. Experimental tests performance

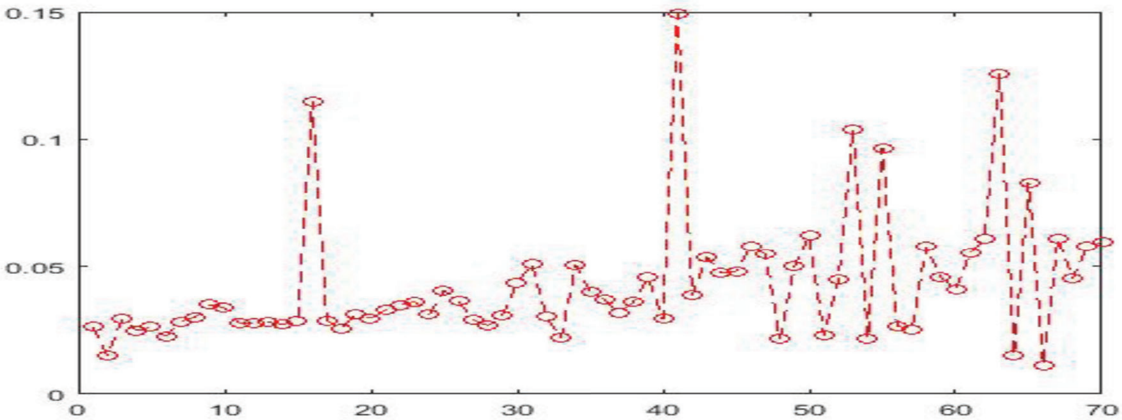
In this section, we propose the results of a symmetric association calculation between two interpolated curves and the original curve, based on a computation of all the distances between each points of these two curves (providing a potential map). In both cases, the mathematical representations that we manipulate are curves that we want to compare two by two. The result is somehow a measure reflecting their similarity.

Figure 6 shows the performance results for the experimental tests by applying the two interpolation types proposed to fingerprint curves having a different arc length (from 5 to 482 points). Figure 6.a shows the RMS error measure that reflects the distance between the reference curves points and the interpolated curves points by Bézier curve method. In the same context, Figure 6.b shows the RMS error measure for cubic splines interpolation.

Fig. 6.   RMS error: (a) Bézier curve method error, (b) cubic spline method error



a



The obtained results reveal that the cubic spline method generates an error slightly higher than that produced by the Bézier curve method. Indeed, a value about 0.37 pixels is reached for the average maximum error and a value about 0.043 pixels is obtained for the average RMS for the cubic spline method. Nevertheless, the Bézier curve method is more powerful than the other proposed methods using the above criteria. Actually, the average maximum error is in the order of 0.33 pixels and the average RMS value is equal to 0.035 pixel.

6.4. Processing time:

The algorithm described in this document takes a lower time of one second to interpolate a single curve. Indeed, the execution time related to the Bézier curve method is about 56 seconds for 70 curves and 58 seconds for the same curve number using the cubic spline method.

7. Discussion

In this section, we compare our numerical results with the results found in other studies. For the method proposed by Aylward and Bullitt, 2002, it has a mean error less than 0.5 voxel. Cavinato et al., 2013 suggested a method characterized by an error on the centre-line of 0.62 ± 0.17 voxel. On the other hand, the method proposed by Guedri et al., 2017 has an average effective value about 0.12 pixels for the cubic spline method, about 0.26 pixels for the linear spline and 0.35 pixels for the least square. On the contrary, the method proposed in this work gives better result in terms of error It has an average RMS error about 0.035 pixels for the Bézier curve method and 0.043 for the cubic spline method, which reveals the reliability and validity of the proposed method. Furthermore, it gives a smooth curve closest to the initial curve and the closest to reality. It also makes these curves natural with the minimum error.

8. Conclusion

Fingerprint based biometric systems involve image enhancement and minutiae extraction as the most commonly used technique. This paper presents a review of various techniques to smooth the fingerprint curves in order to make it closer to reality, and allows us to eliminate the noise present in digitized images. The proposed method is based on two major phases. The first phase concerns the pre-treatment and segmentation for the fingerprint image. It is divided into three parts. The first part aims to enhance image with the Gabor filter whereas the second part is devoted to image binarization. The third part deals with the central lines detection from the fingerprint and minutiae extraction. The second phase is devoted to the smoothing technique of these central lines. Two interpolation types, namely Bezier curve and cubic splines, have been investigated. The proposed approach produced results which are very close to reality with very small errors when using these two interpolation types. As indicated above, the average RMS value is about 0.035 pixels and the maximum error value is about 0.33 pixels for the Bezier curve method. However, the cubic spline method has an average RMS value of 0.043 pixels and a maximum error of 0.37 pixels.

Conflict of Interest

The authors declare no conflict of interest.

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