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Heat Transfer from Moving Surfaces in a Micropolar Fluid with Internal Heat Generation

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Abstract

A boundary layer analysis is presented for the forced convection problem of a surface moving continuously in a flowing stream of a micropolar with internal heat generation. Two cases are considered, one corresponding to a plane surface moving in parallel with the free-stream and the other, a surface moving in the opposite direction to the free stream. A two-dimensional similarity solution to the governing equations momentum, angular momentum and energy is derived. Numerical data for the local friction coefficients, local Nusselt number and local wall couple stress has been shown tabulated for a range of values of material properties, micro-polar parameter Δ and (with and without) heat generation.

Keywords: Heat transfer, Moving surfacees, Micropolar fluid, Internal heat generation.
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1. Introduction

A heat transfer from a moving surface with exponential internal heat generation is of interest in polymer extrusion processes. Two types of classical forced convection problems have been studied extensively in the literature of boundary layer in fluid, fluid with less friction Blasius problems 1908. steady flow over a stationary flat plate Liu et. al 1987; and the Sakiadis problem 1961 of the plate moving continuously in a quiescent ambient fluid. However, in many practical engineering systems, both the plane surface and the ambient fluid are moving in parallel Tsou 1967. The problem of flow and heat transfer of plane surfaces moving in parallel and in the opposite direction to the free stream has been investigated by Lin and Huang 1994. These studies were concerned with the dynamics of Newtonian fluids without heat generation.

A new stage in the evolution of fluid dynamic theory is in progress because of the increasing importance, in the processing industries and elsewhere, of materials whose flow behavior in shear cannot be characterized by Newtonian relationships. Eringen 1966 formulated the theory can be used to explain the flow of colloidal fluid, liquid crystals, animal blood, etc, Eringen 1972 extended the micro-polar fluid theory and developed the theory of thermo-micro-polar fluids. Gorla 1980 studied the thermal boundary layer of a micro-polar fluid at stagnation point using Eringen's theory of micro-polar fluids 1972. Jena and Mathur 1981 obtained a similarity solution for laminar free-convective flow of a thermomicropolar fluid along a vertical plate. Rees and Basson 1996 studied the Blasius boundary layer flow of a micropolar fluid. Mansour and Gorla 1999 studied micropolar fluid past a continuously moving plate in the presence of magnetic field. Effect of suction-injection on the flow of a micropolar fluid past a continuously moving plate in presence of radiation studied by El-Arabawy 2003. Similarity solutions of natural convection with internal heat generation studied by Crepeau and Clarksean 1997. El-Hakiem and Gorla 1999 studied natural convection in a micropolar fluid with internal heat generation. El-Hakiem et.al 2000 studied natural convection from combined thermal and mass diffusion buoyancy boundary effects in micropolar fluids.

Natural convection in a micropolar fluid with thermal dispersion and internal heat generation investigate by El-hakiem 2004. M. Rahman, et al. 2006 studied MHD convective flow of a micropolar fluid past a continuously moving vertical porous plate in presence of heat generation. El-Hakiem et al. 2007 studied combined heat and mass transfer on non-Darcy natural convection in a fluid saturated porous medium with themophoresis. Makinde 2011, find similarity solution for natural convection from a moving vertical plate with internal heat generation and convective boundary condition.

We present here a similarity analysis for the forced convection problem of a surface moving continuously in a flowing stream of a micropolar fluid with heat generation. We study not only the case of a plane surface moving in parallel with the free stream but also the case of a surface moving opposite direction. Numerical results are presented for any velocity ratio of the surface to the free steam and range of values of material parameters of the fluid and internal heat generation are considered.

2. BASIC EQUATIONS

Consider a plane surface moving at a constant velocity u_w in parallel or in the opposite direction to a free stream of a micropolar fluid of uniform velocity u_∞. Either the surface velocity or the free-stream velocity may be zero but not both at the same time. The physical properties of the fluid are assumed to be constant. The governing equations of the steady, laminar bounder-layer flow on the moving surface under these conditions are:

Mass equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

Momentum equation:

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = (\nu + \frac{K}{\rho})\frac{\partial^2 u}{\partial y^2} + \frac{K}{\rho}\frac{\partial N}{\partial y} \qquad (2)$$

Angular momentum equation:

$$u\frac{\partial N}{\partial x} + v\frac{\partial N}{\partial y} = \frac{\gamma}{\rho j}\frac{\partial^2 N}{\partial y^2} - \frac{K}{\rho j}(2N + \frac{\partial u}{\partial y}) \qquad (3)$$

Energy equation:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\nu}{Pr}\frac{\partial^2 T}{\partial y^2} + A \qquad (4)$$

where u and v are velocity components associated with x and y the directions measured along and normal to the vertical plate, respectively; N the angular velocity; K the vortex viscosity; γ the spin gradient viscosity; j the micro-inertia per unit mass; ρ the density of the fluid; ν the kinematic coefficient of viscosity; the internal volumetric heat generation (

$A = \dfrac{\alpha(Re_w + Re_\infty)(T_w - T_\infty)}{x^2} e^{-\eta}$) and $\Pr$ the Prandtl number.

The boundary conditions of this system are:

$$y = 0, u = \pm u_w, v = 0, N = 0, T = T_w,$$

$$y \to \infty : u = u_\infty, N = 0, T = T_\infty. \qquad (5)$$

the boundary condition $u = +u_w$ in (5) represents the case of a plane surface moving in parallel to the free stream, while $u = -u_w$ reorients the case of a surface moving in the opposite direction. To analyze the effect of both the moving surface and the free stream on the boundary-layer flow, we propose a new similarity coordinate and a dimensionless stream function

$$\eta = \frac{y}{x}(\mathrm{Re}_w + \mathrm{Re}_\infty)^{1/2},$$

$$N = \frac{\nu}{x^2}\left[\mathrm{Re}_w + \mathrm{Re}_\infty\right]^{3/2} g(\eta)$$

$$T = T_\infty + (T_w - T_\infty)\theta(\eta). \qquad (6)$$

which are the combinations of the traditional ones

$$\eta_B = \frac{y}{x}\mathrm{Re}_\infty^{1/2}, \; f_B = \frac{\psi}{\nu \mathrm{Re}_\infty^{1/2}} \qquad (7)$$

for the Blasius problem; and

$$\eta_S = \frac{y}{x}\mathrm{Re}_w^{1/2}, \qquad f_S = \frac{\psi}{\nu\,\mathrm{Re}_w^{1/2}} \quad (8)$$

for the Sakiadis problem, where the Reynolds

number are: $\mathrm{Re}_w = \dfrac{u_w x}{\nu}, \quad \mathrm{Re}_\infty = \dfrac{u_\infty x}{\nu}$ (9)

A velocity ratio parameter γ_1 is defined as:

$$\gamma_1 = \frac{u_w}{(u_w + u_\infty)} = \left[1 + \frac{u_\infty}{u_w}\right]^{-1} = \left[1 + \frac{\mathrm{Re}_\infty}{\mathrm{Re}_w}\right]^{-1} \quad (10)$$

Note that for the Blasius problem, $u_w = 0$ therefore $\gamma_1 = 0$. On the other hand, for the Sakiadis problem, $u_\infty = 0$, and thus $\gamma_1 = 1$. We define a dimensionless angular velocity and temperature functions as

$$N = \frac{\nu}{x^2}(Re_w + Re_\infty)^{3/2} g(\eta),$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \quad (11)$$

Under the present transformation variables defined in (6)-(11), the governing transformed equations may be written as

$$(1+\Delta) f''' + \Delta g' + \frac{1}{2} f f'' = 0 \quad (12)$$

$$\lambda g'' - \Delta.B(2g + f'') + \frac{1}{2}(g' f + g f') = 0 \quad (13)$$

$$\frac{1}{\mathrm{Pr}}[\theta'' + c^* e^{-\eta}] + \frac{1}{2} f\theta' = 0 \quad (14)$$

where ($c^* = 1$ with heat generation and $c^* = 0$ without heat generation).

Since the momentum equation (2) is nonlinear, a simple change of coordinates from a rest trams to a moving frame cannot yield a correct solution to the problem under consideration. The proposed similarity transformation is different from the previous analyses in the following ways:

The stationary plate problem (Blasius problem) and moving plate problem (Sakiadis problem) may be obtained as special cases of the transformation given in this paper by setting $\gamma_1 - 1 = 0$ and 1, respectively. The transformation given in this paper has never been applied before to study micropolar fluid boundary layers. The boundary conditions are given by:

$$f(0) = 0, \ f'(0) = \pm\gamma_1, \quad g(0) = 0, \theta(0) = 1,$$
$$f'(\infty) = 1 - \gamma_1, \ g(\infty) = 0, \ \theta(\infty) = 0 \quad (15)$$

In these equations, a prime denotes differentiation with respect to η. We define:

$$\Delta = \frac{K}{\nu\rho}, \quad \lambda = \frac{\gamma}{\rho j \nu}, \quad B = \frac{x^2}{j(Re_w + Re_\infty)^2}$$

and $c^* = \dfrac{(Re_w + Re_\infty)(T_w - T_\infty)}{x^2}$ (16)

The wall shear stress is related to $f''(0)$ by the relation

$$\tau_w = (\nu\rho + K)\left|\left[\frac{\partial u}{\partial y}\right]_{y=0}\right|$$
$$+ K\left|N_{y=0}\right| = \rho(\frac{\nu}{x})^2(1+\Delta)(\mathrm{Re}_w + \mathrm{Re}_\infty)^{3/2}\left|f''(0)\right| \quad (17)$$

To express the wall shear stress non-dimensionally, we define two types of the local friction coefficients

as: $C_\infty = \dfrac{2\tau_w}{\rho u_\infty^2}, \ C_w = \dfrac{2\tau_w}{\rho u_w^2}$ (18)

A combination of (16) and (17) gives $C_\infty Re_\infty^{1/2} = 2(1+\Delta)(1-\gamma_1)^{-3/2}|f''(0)|$, $C_w Re_w^{1/2} = 2(1+\Delta)\gamma_1^{-3/2}|f''(0)|$. (19)

The wall couple stress is related to $g'(0)$ by the relation

$$M_w = \gamma\left|\left[\frac{\partial N}{\partial y}\right]_{y=0}\right| = \gamma(\frac{\nu}{x^2})(Re_w + Re_\infty)^2|g'(0)|$$ (20)

The local Nusselt number, $Nu = \frac{hx}{k}$, can be obtained from the numerical results by the relation

$$\frac{Nu}{Re_\infty^{1/2}} = -\theta'(0). \quad (21)$$

3. Results and Discussion

The resulting ordinary differential Eqs. (12)-(14) with corresponding boundary conditions (15) are solved by numerical integration using the fourth-order Runge-Kutta method and Newton Raphson technique by giving proper guess values for $f''(0)$, $\theta'(0)$ and $g'(0)$. The present results are accurate upto sixth decimal place. Numerical computations are carried out on computer for $Pr = 0.7$, $B = 0.1$ and $\lambda = 0.5$, where Δ , γ_1 were varied over a range. The accuracy of the numerical results has been verified by comparing the present data for a Newtonian fluid and without heat generation ($\Delta = 0$, $c^* = 0$) with those Lin and Huang [5].

Tables 1 and 2 contains a summary of numerical results. Table 1 shows the surface values of velocity gradients, temperature gradients and the micro-rotation components that are proportional to the friction factor, Nusselt number, and wall couple stress, respectively, for the case of a plane surface moving in parallel to the free stream with $c^* = 0$ and $c^* = 1$ (without and with heat generation). Table 2 represents the case of a surface moving in the opposite direction for (without and with heat generation). These Tables illustrate the effects of the velocity ratio γ_1 and micro-polar parameter Δ on the friction factor, Nusselt number, and wall couple stress for without and with heat generation ($c^* = 0$ and $c^* = 1$) respectively.

4. Results presented in Tables 1 and 2 indicate that micropolar fluids display drag and heat transfer rate reduction characteristics for $c^* = 0$ and $c^* = 1$. No effect of heat generation on velocity, angular velocity, friction factor and wall couple stress.The significance of the present work lies in its application in heat transfer augmentation or reduction processes. Tables 1, 2 shows that the heat transfer rate in the case of moving plate is higher than in the case of a stationary plate. In a micropolar fluid applications involving either parallel flow or reverse flow cases, one can choose the proper speed of the plate (γ_1 to obtain an appropriate augmentation and (or) reduction in heat transfer rate.) with $c^* = 0$ and $c^* = 1$. The results from Tables 1, 2 indicate that when $\Delta = 5$ and $\gamma_1 < 0.5$, heat transfer rate is reduced when compared with Newtonian fluids ($\Delta = 0$). For the same value of Δ , when $\gamma_1 > 0.5$, the heat transfer rate increases when compared with $\Delta = 0$ and $c^* = 0$, and $c^* = 1$

Figures 1-3 display the results for the variation of the boundary layer profiles for the velocity, temperature and angular velocity for the case of parallel moving surface. From these figures we observe that the temperature distribution becomes more uniform as the velocity ratio decreases for without and with heat generation. The angular velocity changes sign from negative to positive as the velocity ratio increases. amplitude of the angular velocity enhancis with the velocity ratio. We have used the velocity, the ratio γ_1 and micro-polar parameter Δ to be prescribable parameters. The momentum boundary layer increases with γ_1 and $c^* = 0$(without heat generation). The temperature becomes more uniform as γ_1 increases with $c^* = 0$ and $c^* = 1$.

Table 1. Variation of $f''(0)$, $\theta'(0)$ and $g'(0)$ for the plane surface movingparallel to the free stream

		$c^* = 0$			$c^* = 1$
Δ	γ_1	$f''(0)$	$-\theta'(0)$	$g'(0)$	$\theta'(0)$
0.0	0.0	0.33206	0.29271	0.0	0.31081
	0.1	0.27828	0.30317	0.0	0.30226
	0.3	0.15017	0.32052	0.0	0.29049
	0.5	0.00000	0.33405	0.0	0.28479
	0.7	-0.16772	0.34433	0.0	0.28351
	0.9	-0.34944	0.35158	0.0	0.28101
	1.0	-0.44326	0.35412	0.0	0.27813
0.5	0.0	0.26176	0.27690	-0.06390	0.33391
	0.1	0.22029	0.29039	-0.05291	0.32104
	0.3	0.11958	0.31392	-0.02783	0.30033
	0.5	0.00000	0.33405	0.000000	0.28479
	0.7	-0.13456	0.35163	0.02959	0.27346
	0.9	-0.28130	0.36711	0.06029	0.26592
	1.0	-0.35855	0.37417	0.07587	0.26352
1.5	0.0	0.18599	0.26005	-0.10025	0.35862
	0.1	0.15760	0.27700	-0.08617	0.34078
	0.3	0.08644	0.30720	-0.04809	0.31038
	0.5	0.00000	0.33405	0.000000	0.28479
	0.7	-0.09870	0.35863	0.05551	0.26261
	0.9	-0.20764	0.38154	0.11661	0.24308
	1.0	-0.26548	0.39249	0.14881	0.23418
5.0	0.0	0.11266	0.23747	-0.11379	0.39280
	0.1	0.09496	0.25896	-0.09937	0.36820
	0.3	0.05179	0.29827	-0.05757	0.32408
	0.5	0.00000	0.33405	0.000000	0.28479
	0.7	-0.05901	0.36733	0.07177	0.24886
	0.9	-0.12429	0.39873	0.15636	0.21544
	1.0	-0.15905	0.41386	0.20307	0.19948

Table 2. Variation of $f''(0)$, $\theta'(0)$ and $g'(0)$ for various values of Δ, γ_1 with $c^* = 0$ and $c^* = 1$

		$c^* = 0$			$c^* = 1$
Δ	γ_1	$f''(0)$	$-\theta'(0)$	$-g'(0)$	$\theta'(0)$
0.0	0.0	0.33206	0.29271	0.0	0.31081
	0.05	0.30600	0.27160	0.0	0.33906
	0.1	0.27705	0.24811	0.0	0.37064
	0.15	0.24392	0.22175	0.0	0.40714
	0.2	0.20349	0.18925	0.0	0.45209
	0.25	0.14477	0.14286	0.0	0.51835
	0.26	0.11735	0.11669	0.0	0.56131
0.5	0.0	0.26176	0.27690	0.06390	0.33391
	0.05	0.24002	0.25512	0.06012	0.36330
	0.1	0.21590	0.23097	0.05536	0.39622
	0.15	0.18833	0.20340	0.04912	0.43426
	0.2	0.15516	0.17020	0.04041	0.48078
	0.25	0.11129	0.12560	0.02680	0.54459
	0.26	0.09768	0.09957	0.01859	0.59573
1.5	0.0	0.18599	0.26005	0.10025	0.35862
	0.05	0.17014	0.23764	0.09017	0.38909
	0.1	0.15295	0.21292	0.07900	0.42305
	0.15	0.13401	0.18508	0.0655	0.46168
	0.2	0.11281	0.15302	0.04809	0.50668
	0.25	0.08972	0.11625	0.03829	0.55870
	0.26	0.07531	0.09859	0.02998	0.60085
5.0	0.0	0.11266	0.23747	0.11379	0.39280
	0.05	0.10551	0.21530	0.10342	0.42310
	0.1	0.09815	0.19168	0.09321	0.45554
	0.15	0.09065	0.16661	0.08329	0.49005
	0.2	0.08313	0.14040	0.07386	0.52608
	0.25	0.07585	0.11388	0.06520	0.56225
	0.26	0.07093	0.09585	0.05827	0.61533

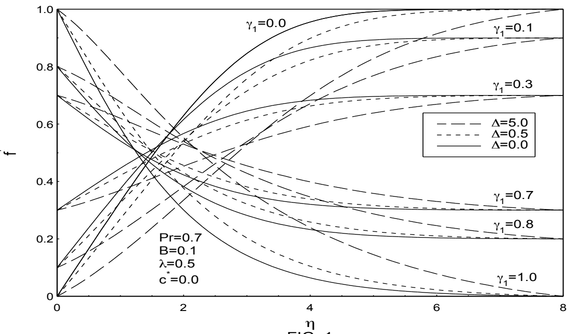

FIG. 1

Fig. 1. Velocity profiles for parallel moving surface, and various Δ, γ_1 with ($c^* = 0$)

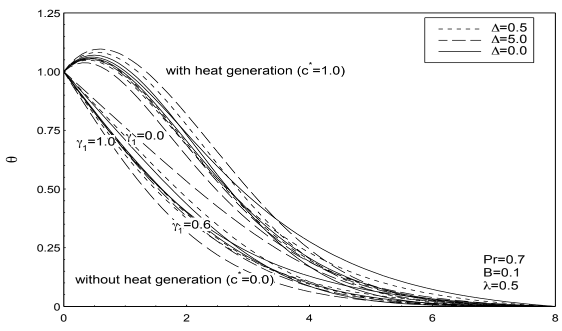

FIG. 2

Fig. 2. Temperature profiles for parallel moving surface, and various Δ, γ_1 with ($c^* = 0.0,1.0$).

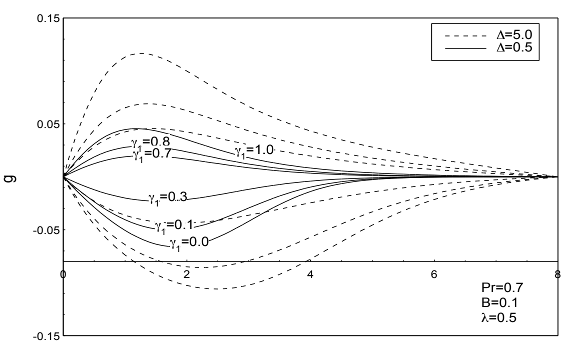

FIG. 3

Fig. 3. Microrotation profiles for parallel moving surface, and various Δ, γ_1 with ($c^* = 0$).

Concluding Remarks:

Similarity solutions have been developed to analyzed the effect of an exponential form for the heat generation for the problem of a continuous moving surface in a micropolar flowing fluid by introducing novel transformation variable and parameters of velocity ratio. For the case of a plane surface moving in parallel to a free stream, very accurate similarity solutions are obtained for predicting the wall friction, heat transfer, and wall couple stress rate for any ratio of surface velocity and free stream velocity over the range of $0 \le \Delta \le 5$. The case of a surface moving in the reverse direction of the free stream has also been analyzed. Velocity, angular velocity and temperature profiles have been presented to show the effects of micro-polar parameter Δ and the relative motion of the plane surface the stream (with and without heat generation).

Nomenclature

A =a constant in equation (4)

f = non-dimensional stream function
g = non-dimensional micro-rotation
h = local heat transfer coefficient
j = microinertia per unit mass
K = vortex viscosity
k = thermal conductivity of fluid
M_w = local couple stress
N = angular velocity
Nu = local Nusselt number
$\Pr$ = Prandtl number
T = temperature
u = vertical velocity component
v = horizontal velocity component
x = vertical coordinate
y = horizontal coordinate
γ = spin gradient viscosity
γ_1 = velocity ratio
Δ, λ = dimensionless material properties
θ = temperature excess ratio
μ = dynamic viscosity
ν = kinematic viscosity
ρ = density
τ_w = shear stress
ψ = stream function

Superscripts

$'$ Differentiation with respect to η

Subscripts

w Refers to conditions at the wall

∞ Refers to conditions far away from the wall

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